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DOMAIN PAIR SYMMETRY REDUCTION DUE TO DISORIENTATIONS[i]

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We tabulate how the symmetry characteristics of ferroelastic domain pairs are influenced by disorientations, i.e. rotations of single domain states needed to achieve a coherent interface of two ferroelastic domain states along a planar wall.

Keywords: domain pair, symmetry, disorientations

INTRODUCTION

In predicting the tensor distinction of two domain states by group theoretical means the concept of the *symmetry group of a domain pair* has been introduced [1]: Let S_i denote a single domain state, i.e. a bulk of one single domain; single domain states have the same structure, differ only in their spatial orientations and are related by symmetry operations of the parent symmetry group. Two domain states S_1 and S_2 ,considered irrespective of their coexistence, form a *domain pair* $\{S_1, S_2\}$. Group theoretically, the domain states of a domain pair are specified by the symmetry group $\mathbf{F_1}$ of S_1 , the subgroup of all symmetry operations of the parent symmetry group which leaves the first domain state S_1 invariant and a switching operation g_{12} that

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transforms S_1 into the second domain state S_2 , i.e. $g_{12}S_1 = S_2$. The symmetry group $\mathbf{F_2}$ of the second domain state is given by $\mathbf{F_2} = g_{12} \mathbf{F_1} g_{12}^{-1}$. Let $\mathbf{F_{12}} = \mathbf{F_1} \cap \mathbf{F_2}$ denote the group of all symmetry elements which leave both domain states S_1 and S_2 invariant and let $j_{12}*$ denote a symmetry operation, if one exists, that interexchanges the two domain states, i.e. $j_{12}*S_1 = S_2$ and $j_{12}*S_2 = S_1$. The symmetry group of a domain pair $\{S_1, S_2\}$ is given by $\mathbf{J_{12}} = \mathbf{F_{12}} + j_{12}*\mathbf{F_{12}}$. If there exists an element $j_{12}*$ then the domain pair $\{S_1, S_2\}$ is referred to as a *transposable* domain pair. If in addition $\mathbf{F_1} = \mathbf{F_2}$, it is referred to as a *completely transposable* domain pair. If no such element exists, then it is referred to as a *non-transposable* domain pair.

For non-ferroelastic domain pairs, the mutual orientations of the domain states in a polydomain are not influenced by conditions of coexistence. The domain states in a polydomain sample are identical with single domain states. All possible non-ferroelastic domain pairs are completely transposable [2,3]. For ferroelastic domain pairs, the mutual orientations of the domain states in a polydomain sample are influenced by conditions of coexistence and their orientation is not identical with that of single domain states. It has been shown [4-6], that for a ferroelastic domain pair there may exist two mutually perpendicular planes, called *permissible walls*, along which two domain states can meet in a compatible manner, i.e. without dislocations or other singular defects. To achieve such a compatible interface the two single domain states must be rotated by a *disorientation angle* $\phi/2$ and $-\phi/2$, respectively, about the intersection of the permissible domain walls called an axis of the *ferroelastic domain pair axis*. The rotated domain states are called *disoriented* and a domain pair consisting of two disoriented domain states with a common permissible wall, a *compatible domain pair*.

DOMAIN PAIR SYMMETRY REDUCTION DUE TO DISORIENTATIONS

The symmetry characteristics of ferroelastic compatible domain pairs change due to the disorientation rotation of the constituent domain states necessary for a compatible interface. In Table 1 we list these symmetry characteristics of the domain states and of the domain pair prior and subsequent to the disorientation rotation. We consider all classes of compatible domain pairs [7]. This includes the cases of completely transposable ferroelastic compatible domain pairs [8].

For each class of ferroelastic compatible domain pairs two lines of symmetry characteristics are given, the first for prior to and the second for subsequent to the disorientation rotation. Prior to the disorientation rotation (first line):

F_1 The symmetry group of the first domain state S_1.

g_{12} The switching operation, $g_{12} S_1 = S_2$. If available, two choices are given.

K_{12} The twinning group, the group $< \mathbf{F_1} , g_{12} >$ generated by the symmetry group $\mathbf{F_1}$ of the first domain state S_1 and the switching operation g_{12} .

F_2 The symmetry group of the second domain state S_2 .

F_{12} The symmetry group common to both domain states, i.e. the intersection $\mathbf{F_1} \cap \mathbf{F_2}$ of the symmetry groups of the two domain states.

j_{12}* The twinning operation, an operation that switches the two domain states, $j_{12}^*S_1 = S_2$ and $j_{12}^*S_2 = S_1$. If none exist an entry "none" is given. The "*" denotes an operation which switches the domain states.

J_{12} The symmetry group of the domain pair, the group $\mathbf{F_{12}} + j_{12}^* \mathbf{F_{12}}$.

axis The direction of the axis of the disorientation rotation.

Subsequent to the disorientation rotation (second line): The superscript "+ -" denotes that the symmetry characteristic is that of the domain state or states after the disorientation rotation.

F_1^{+-} The symmetry group of the first domain state S_1^+, the domain state S_1 rotated through $+\phi/2$ about the rotation axis , where ϕ is the disorientation rotation angle (see example in Figures 1 and 2 below). This is defined as the maximal subgroup of $\mathbf{K_{12}}$ which leaves S_1^+ invariant. $\mathbf{F_1}^{+-}$ is also the symmetry group of S_1^- , the domain state S_1 rotated through $-\phi/2$.

The domain pair will consist of either S_1^+ and S_2^- or S_1^- and S_2^+ as the pair of domain states S_1 and S_2 must be rotated in opposite directions to meet at a compatible domain wall.

g_{12}^{+-} The switching operation, $g_{12}^{+-} S_1^+ = S_2^-$ (and $g_{12}^{+-} S_1^- = S_2^+$). If available, two choices are given, if none exist an entry “none” is given.

$\mathbf{K_{12}}^{+-}$ The twinning group, the group < $\mathbf{F_1}^{+-}$, g_{12}^{+-} >.

$\mathbf{F_2}^{+-}$ The symmetry group of the second domain state S_2^- (and S_2^+).

$\mathbf{F_{12}}^{+-}$ The symmetry group common to both domain states S_1^+ and S_2^- (and to both the domain states S_1^- and S_2^+).

$j_{12}*^{+-}$ The twinning operation $j_{12}*^{+-}S_1^+ = S_2^-$ and $j_{12}*^{+-}S_2^- = S_1^+$ ($j_{12}*^{+-}S_1^- = S_2^+$ and $j_{12}*^{+-}S_2^+ = S_1^-$). If none exist an entry “none” is given

As an example consider the following derivation of a ferroelastic domain twin from two single domain states. In the Figure 1, the parent phase is represented by a dashed square. The two single domain states S_1 and S_2 are shown as solid rectangles. A double-dashed line “ = = = “ denotes one of the two perpendicular compatible domain walls, both perpendicular to the plane of the paper, and the disorientation angle ϕ is also shown.

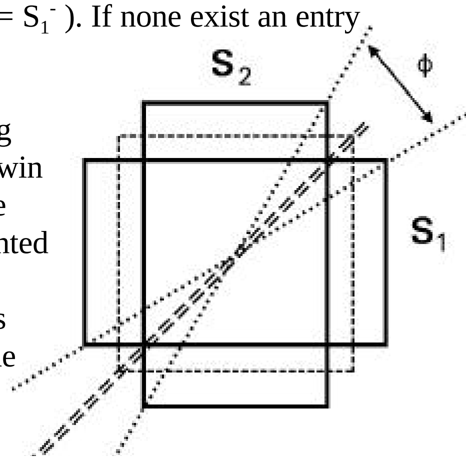

Figure 1

In Figure 2, we show the two single domain states after the disorientation. Domain state S_1 has been rotated -ϕ/2 into the domain state S_1^- and S_2 into S_2^+. Assuming that prior to the disorientation, $F_1 = \mathbf{2_x 2_y 2_z}$ and $g_{12} = 2_{xy}$, one has $\mathbf{K_{12} = 4_z 2_x 2_{xy}}$, and the domain pair symmetry group $\mathbf{J_{12} = 4_z^* 2_x 2_{xy}}$. From Table 1, we then have that after the disorientation, we find the reduction in symmetries

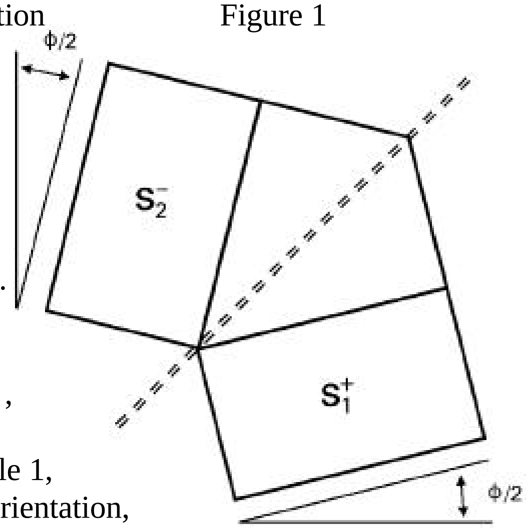

Figure 2

to be $\mathbf{F_1}^{+-} = \mathbf{2_z}$, $g_{12}^{+-} = 2_{xy}$, and $\mathbf{J_{12}}^{+-} = \mathbf{2_{xy}{*}2_{\bar{x}y}{*}2_z}$.

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Table 1: Domain Pair Symmetry Reduction

$\mathbf{F}_1$ $\mathbf{F}_1^{+-}$	g_{12} g_{12}^{+-}	$\mathbf{K}_{12}$ $\mathbf{K}_{12}^{+-}$	$\mathbf{F}_2$ $\mathbf{F}_2^{+-}$	$\mathbf{F}_{12}$ $\mathbf{F}_{12}^{+-}$	$j_{12}*$ $j_{12}*^{+-}$	$\mathbf{J}_{12}$ $\mathbf{J}_{12}^{+-}$	axis
$\mathbf{1}$	2_z	$\mathbf{2_z}$	$\mathbf{1}$	$\mathbf{1}$	2_z*	$\mathbf{2_z*}$	$[k\bar{h}0]$
$\mathbf{1}$	2_z	$\mathbf{2_z}$	$\mathbf{1}$	$\mathbf{1}$	2_z*	$\mathbf{2_z*}$	
$\mathbf{1}$	m_z	$\mathbf{m_z}$	$\mathbf{1}$	$\mathbf{1}$	m_z*	$\mathbf{m_z*}$	$[k\bar{h}0]$
$\mathbf{1}$	m_z	$\mathbf{m_z}$	$\mathbf{1}$	$\mathbf{1}$	m_z*	$\mathbf{m_z*}$	
$\bar{\mathbf{1}}$	2_z , m_x	$\mathbf{2_z/m_z}$	$\bar{\mathbf{1}}$	$\bar{\mathbf{1}}$	2_z*	$\mathbf{2_z*/m_z*}$	$[k\bar{h}0]$
$\bar{\mathbf{1}}$	2_z , m_x	$\mathbf{2_z/m_z}$	$\bar{\mathbf{1}}$	$\bar{\mathbf{1}}$	2_z*	$\mathbf{2_z*/m_z*}$	
$\mathbf{2_z}$	2_x , 2_y	$\mathbf{2_x2_y2_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	2_x*	$\mathbf{2_x*2_y*2_z}$	[001]
$\mathbf{2_z}$	2_x , 2_y	$\mathbf{2_x2_y2_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	2_x*	$\mathbf{2_x*2_y*2_z}$	
$\mathbf{2_z}$	m_x , m_y	$\mathbf{m_xm_y2_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	m_x	$\mathbf{m_x*m_y*2_z}$	[001]
$\mathbf{2_z}$	m_x , m_y	$\mathbf{m_xm_y2_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	m_x	$\mathbf{m_x*m_y*2_z}$	
$\mathbf{2_z}$	4_z , 4_z^3	$\mathbf{4_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	4_z	$\mathbf{4_z*}$	[001]
$\mathbf{2_z}$	none	$\mathbf{2_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	none	$\mathbf{2_z}$	
$\mathbf{2_z}$	$\bar{4}_z$, $\bar{4}_z^3$	$\bar{\mathbf{4}}_\mathbf{z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	$\bar{4}_z$	$\bar{\mathbf{4}}_\mathbf{z}\mathbf{*}$	[001]
$\mathbf{2_z}$	none	$\mathbf{2_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	none	$\mathbf{2_z}$	
$\mathbf{2_x}$	2_{xy} , 4_z	$\mathbf{4_z2_x2_{xy}}$	$\mathbf{2_y}$	$\mathbf{1}$	2_{xy}	$\mathbf{2_{xy}*}$	$[\bar{l}l2h]$
$\mathbf{1}$	2_{xy}	$\mathbf{2_{xy}}$	$\mathbf{1}$	$\mathbf{1}$	2_{xy}	$\mathbf{2_{xy}*}$	
$\mathbf{2_x}$	m_{xy} , $\bar{4}_z$	$\bar{\mathbf{4}}_\mathbf{z}\mathbf{2_xm_{xy}}$	$\mathbf{2_y}$	$\mathbf{1}$	m_{xy}	$\mathbf{m_{xy}*}$	$[\bar{l}l2h]$
$\mathbf{1}$	m_{xy}	$\mathbf{m_{xy}}$	$\mathbf{1}$	$\mathbf{1}$	m_{xy}	$\mathbf{m_{xy}*}$	
$\mathbf{2_x}$	$2_{x'}$, 3_z^2	$\mathbf{3_z2_x}$	$\mathbf{2_{x''}}$	$\mathbf{1}$	$2_{x'}*$	$\mathbf{2_{x'}*}$	$[l0\bar{l}2h]$
$\mathbf{1}$	$2_{x'}$	$\mathbf{2_{x'}}$	$\mathbf{1}$	$\mathbf{1}$	$2_{x'}*$	$\mathbf{2_{x'}*}$	
$\mathbf{2_x}$	$m_{x'}$, $\bar{3}_z^5$	$\bar{\mathbf{3}}_\mathbf{z}\mathbf{m_x}$	$\mathbf{2_{x''}}$	$\mathbf{1}$	$m_{x'}*$	$\mathbf{m_{x'}*}$	$[l0\bar{l}2h]$
$\mathbf{1}$	$m_{x'}$	$\mathbf{m_{x'}}$	$\mathbf{1}$	$\mathbf{1}$	$m_{x'}*$	$\mathbf{m_{x'}*}$	
$\mathbf{2_z}$	3_z^2 , 6_z	$\mathbf{6_z}$	$\mathbf{2_z}$	$\mathbf{2_z}$	none	$\mathbf{2_z}$	[0001]

DOMAIN PAIR SYMMETRY REDUCTION

2_z	none	2_z	2_z	2_z	none	2_z	
2_z	$\bar{3}_z^2$, $\bar{6}_z$	$6_z/m_z$	2_z	2_z	none	2_z	[0001]
2_z	none	2_z	2_z	2_z	none	2_z	
2_x	$2_{y'}$, 6_z	$6_z2_x2_y$	$2_{x''}$	1	$2_{y'}$*	$2_{y'}$*	[l 2$\bar{l}$ l2h]
1	$2_{y'}$	$2_{y'}$	1	1	$2_{y'}$*	$2_{y'}$*	
2_x	$m_{y'}$, $\bar{6}_z$	$\bar{6}_z2_xm_y$	$2_{x''}$	1	$m_{y'}$*	$m_{y'}$*	[l 2$\bar{l}$ l 2h]
1	$m_{y'}$	$m_{y'}$	1	1	$m_{y'}$*	$m_{y'}$*	
m_z	2_y , m_x	$m_x2_ym_z$	m_z	m_z	2_y*	m_x*2_y*m_z	[001]
m_z	2_y , m_x	$m_x2_ym_z$	m_z	m_z	2_y*	m_x*2_y*m_z	
m_z	4_z , $\bar{4}_z^3$	$4_z/m_z$	m_z	m_z	none	m_z	[001]
m_z	none	m_z	m_z	m_z	none	m_z	
m_x	m_{xy} , 4_z	$4_zm_xm_{xy}$	m_y	1	m_{xy}*	m_{xy}*	[$\bar{l}$ l2h]
1	m_{xy}	m_{xy}	1	1	m_{xy}*	m_{xy}*	
m_x	2_{xy} , $\bar{4}_z$	$\bar{4}_zm_x2_{xy}$	m_y	1	2_{xy}*	2_{xy}*	[$\bar{l}$ l2h]
1	2_{xy}	2_{xy}	1	1	2_{xy}*	2_{xy}*	
m_x	$m_{x'}$, 3_z^2	3_zm_x	$m_{x''}$	1	$m_{x'}$*	$m_{x'}$*	[l 0$\bar{l}$ 2$\bar{h}$]
1	$m_{x'}$	$m_{x'}$	1	1	$m_{x'}$*	$m_{x'}$*	
m_x	$2_{x'}$, $\bar{3}_z^2$	$\bar{3}_zm_x$	$m_{x''}$	1	$2_{x'}$*	$2_{x'}$*	[l 0$\bar{l}$ 2$\bar{h}$]
1	$2_{x'}$	$2_{x'}$	1	1	$2_{x'}$*	$2_{x'}$*	
m_z	3_z , $\bar{6}_z^5$	$\bar{6}_z$	m_z	m_z	none	m_z	[0001]
m_z	none	m_z	m_z	m_z	none	m_z	
m_z	$\bar{3}_z^2$, 6_z	$6_z/m_z$	m_z	m_z	none	m_z	[0001]
m_z	none	m_z	m_z	m_z	none	m_z	
m_x	$m_{y'}$, 6_z	$6_zm_xm_y$	$m_{x''}$	1	$m_{y'}$*	$m_{y'}$*	[l 2$\bar{l}$ l2h]
1	$m_{y'}$	$m_{y'}$	1	1	$m_{y'}$*	$m_{y'}$*	
m_x	$2_{y'}$, $\bar{6}_z$	$\bar{6}_zm_x2_y$	$m_{x''}$	1	$2_{y'}$*	$2_{y'}$*	[l 2$\bar{l}$ l2h]
1	$2_{y'}$	$2_{y'}$	1	1	$2_{y'}$*	$2_{y'}$*	
$2_z/m_z$	m_x , m_y	$m_xm_ym_z$	$2_z/m_z$	$2_z/m_z$	m_x*	m_x*m_y*m_z	[001]
$2_z/m_z$	m_x , m_y	$m_xm_ym_z$	$2_z/m_z$	$2_z/m_z$	m_x*	m_x*m_y*m_z	
$2_z/m_z$	4_z , 4_z^3	$4_z/m_z$	$2_z/m_z$	$2_z/m_z$	4_z*	4_z*$/m_z$	[001]
$2_z/m_z$	none	$2_z/m_z$	$2_z/m_z$	$2_z/m_z$	none	$2_z/m_z$	
$2_x/m_x$	m_{xy} , 4_z	$4_z/m_zm_xm_{xy}$	$2_y/m_y$	$\bar{1}$	2_{xy}*	2_{xy}*$/m_{xy}$*	[$\bar{l}$ l2h]
$\bar{1}$	2_{xy} , m_{xy}	$2_{xy}/m_{xy}$	$\bar{1}$	$\bar{1}$	2_{xy}*	2_{xy}*$/m_{xy}$*	
$2_x/m_x$	$m_{x'}$, 3_z^2	$\bar{3}_zm_x$	$2_{x''}/m_{x''}$	$\bar{1}$	$2_{x'}$*	$2_{x'}$*$/m_{x'}$*	[l 0$\bar{l}$ 2$\bar{h}$]
$\bar{1}$	$2_{x'}$, $m_{x'}$	$2_{x'}/m_{x'}$	$\bar{1}$	$\bar{1}$	$2_{x'}$*	$2_{x'}$*$/m_{x'}$*	
$2_z/m_z$	3_z^3 , 6_z	$6_z/m_z$	$2_z/m_z$	$2_z/m_z$	none	$2_z/m_z$	[0001]
$2_z/m_z$	none	$2_z/m_z$	$2_z/m_z$	$2_z/m_z$	none	$2_z/m_z$	
$2_x/m_x$	$m_{y'}$, 6_z	$6_z/m_zm_xm_y$	$2_{x''}/m_{x''}$	$\bar{1}$	$m_{y'}$*	$2_{y'}$*$/m_{y'}$*	[l 2$\bar{l}$ l2h]
$\bar{1}$	$2_{y'}$, $m_{y'}$	$2_{y'}/m_{y'}$	$\bar{1}$	$\bar{1}$	$m_{y'}$*	$2_{y'}$*$/m_{y'}$*	

$2_x2_y2_z$	2_{xy} , $2_{\bar{x}y}$	$4_z2_x2_{xy}$	$2_x2_y2_z$	$2_x2_y2_z$	2_{xy}*	4_z*2_x2_{xy}*	[001]
2_z	2_{xy} , $2_{\bar{x}y}$	$2_{xy}2_{\bar{x}y}2_z$	2_z	2_z	2_{xy}*	2_{xy}*$2_{\bar{x}y}$*2_z	
$2_x2_y2_z$	m_{xy} , $m_{\bar{x}y}$	$\bar{4}_z2_xm_{xy}$	$2_x2_y2_z$	$2_x2_y2_z$	m_{xy}	$\bar{4}_z$*2_xm_{xy}*	[001]
2_z	m_{xy} , $m_{\bar{x}y}$	$m_{xy}m_{\bar{x}y}2_z$	2_z	2_z	m_{xy}	m_{xy}*$m_{\bar{x}y}$*2_z	
$2_x2_y2_z$	$2_{y'}$, $2_{x'}$	$6_z2_x2_y$	$2_{x''}2_{y''}2_z$	2_z	$2_{y'}$*	$2_{x'}$*$2_{y'}$*2_z	[0001]

2_z	$2_{y'}$, $2_{x'}$	$2_{x'}2_{y'}2_z$	2_z	2_z	$2_{y'}*$	$2_{x'}*2_{y'}*2_z$	
$2_x2_y2_z$	$m_{y'}$, $m_{x'}$	$6_z/m_zm_xm_y$	$2_{x''}2_{y''}2_z$	2_z	$m_{y'}*$	$m_{x'}*m_{y'}*2_z$	[0001]
2_z	$m_{y'}$, $m_{x'}$	$m_{x'}m_{y'}2_z$	2_z	2_z	$m_{y'}*$	$m_{x'}*m_{y'}*2_z$	
$2_{xy}2_{\bar{x}y}2_z$	2_{xz} , 4_y	$4_z3_p2_{xy}$	$2_x2_{yz}2_{\bar{y}z}$	1	$2_{xz}*$	$2_{xz}*$	$[l2h\bar{l}]$
1	2_{xz}	2_{xz}	1	1	$2_{xz}*$	$2_{xz}*$	
$2_{xy}2_{\bar{x}y}2_z$	m_{xz} , $\bar{4}_y$	$m_z\bar{3}_pm_{xy}$	$2_x2_{yz}2_{\bar{y}z}$	1	$m_{xz}*$	$m_{xz}*$	$[l2h\bar{l}]$
1	m_{xz}	m_{xz}	1	1	$m_{xz}*$	$m_{xz}*$	
$m_xm_y2_z$	m_{xy} , $m_{\bar{x}y}$	$4_zm_xm_{xy}$	$m_xm_y2_z$	$m_xm_y2_z$	$m_{xy}*$	$4_z*m_xm_{xy}*$	[001]
2_z	m_{xy} , $m_{\bar{x}y}$	$m_{xy}m_{\bar{x}y}2_z$	2_z	2_z	$m_{xy}*$	$m_{xy}*m_{\bar{x}y}*2_z$	
$m_{xy}m_{\bar{x}y}2_z$	2_x , 2_y	$\bar{4}_z2_xm_{xy}$	$m_{xy}m_{\bar{x}y}2_z$	$m_{xy}m_{\bar{x}y}2_z$	2_x*	$\bar{4}_z*2_x*m_{xy}$	[001]
2_z	2_x , 2_y	$2_x2_y2_z$	2_z	2_z	2_x*	$2_x*2_y*2_z$	
$2_xm_ym_z$	2_{xy} , $m_{\bar{x}y}$	$4_z/m_zm_xm_{xy}$	$m_x2_ym_z$	m_z	$2_{xy}*$	$2_{xy}*m_{\bar{x}y}*m_z$	[001]
m_z	2_{xy} , $m_{\bar{x}y}$	$2_{xy}m_{\bar{x}y}m_z$	m_z	m_z	$2_{xy}*$	$2_{xy}*m_{\bar{x}y}*m_z$	
$m_xm_y2_z$	$m_{y'}$, $m_{x'}$	$6_zm_xm_y$	$m_{x''}m_{y''}2_z$	2_z	$m_{y'}*$	$m_{x'}*m_{y'}*2_z$	[0001]
2_z	$m_{y'}$, $m_{x'}$	$m_{x'}m_{y'}2_z$	2_z	2_z	$m_{y'}*$	$m_{x'}*m_{y'}*2_z$	
$m_x2_ym_z$	$2_{y'}$, $m_{x'}$	$\bar{6}_zm_x2_y$	$m_{x''}2_{y''}m_z$	m_z	$2_{y'}*$	$m_{x'}*2_{y'}*m_z$	[0001]
m_z	$2_{y'}$, $m_{x'}$	$m_{x'}2_{y'}m_z$	m_z	m_z	$2_{y'}*$	$m_{x'}*2_{y'}*m_z$	
$2_xm_ym_z$	$2_{y'}$, $m_{x'}$	$6_z/m_zm_xm_y$	$2_{x''}m_{y''}m_z$	m_z	$m_{x'}*$	$m_{x'}*2_{y'}*m_z$	[0001]
m_z	$2_{y'}$, $m_{x'}$	$m_{x'}2_{y'}m_z$	m_z	m_z	$m_{x'}*$	$m_{x'}*2_{y'}*m_z$	
$m_xm_y2_z$	$2_{y'}$, $2_{x'}$	$6_z/m_zm_xm_y$	$m_{x''}m_{y''}2_z$	2_z	$2_{y'}*$	$2_{x'}*2_{y'}*2_z$	[0001]
2_z	$2_{y'}$, $2_{x'}$	$2_{x'}2_{y'}2_z$	2_z	2_z	$2_{y'}*$	$2_{x'}*2_{y'}*2_z$	
$m_{xy}m_{\bar{x}y}2_z$	m_{xz} , $\bar{4}_y$	$\bar{4}_z3_pm_{xy}$	$2_xm_{yz}m_{\bar{y}z}$	1	$m_{xz}*$	$m_{xz}*$	$[l2h\bar{l}]$
1	m_{xz}	m_{xz}	1	1	$m_{xz}*$	$m_{xz}*$	
$m_{xy}m_{\bar{x}y}2_z$	2_{xz} , 4_y	$m_z\bar{3}_pm_{xy}$	$2_xm_{yz}m_{\bar{y}z}$	1	$2_{xz}*$	$2_{xz}*$	$[l2h\bar{l}]$
1	2_{xz}	2_{xz}	1	1	$2_{xz}*$	$2_{xz}*$	
$2_{xy}m_{\bar{x}y}m_z$	2_{xz} , $\bar{4}_y$	$m_z\bar{3}_pm_{xy}$	$m_xm_{yz}2_{\bar{y}z}$	1	$2_{xz}*$	$2_{xz}*$	$[l2h\bar{l}]$
1	2_{xz}	2_{xz}	1	1	$2_{xz}*$	$2_{xz}*$	
$2_{xy}m_{\bar{x}y}m_z$	m_{xz} , 4_y	$m_z\bar{3}_pm_{xy}$	$m_xm_{\bar{y}z}2_{yz}$	1	$m_{xz}*$	$m_{xz}*$	$[l2h\bar{l}]$
1	m_{xz}	m_{xz}	1	1	$m_{xz}*$	$m_{xz}*$	
$m_xm_ym_z$	$2_{\bar{x}y}$, m_{xy}	$4_z/m_zm_xm_{xy}$	$m_xm_ym_z$	$m_xm_ym_z$	$m_{xy}*$	$4_z*/m_zm_xm_{xy}*$	[001]
m_z	$2_{\bar{x}y}$, m_{xy}	$m_{xy}2_{\bar{x}y}m_z$	m_z	m_z	$m_{xy}*$	$m_{xy}*m_{\bar{x}y}*m_z$	
$m_xm_ym_z$	$m_{y'}$, $m_{x'}$	$6_z/m_zm_xm_y$	$m_{x''}m_{y''}m_z$	$2_z/m_z$	$m_{y'}*$	$m_{x'}*m_{y'}*m_z$	[0001]
$2_z/m_z$	$m_{y'}$, $m_{x'}$	$m_{x'}m_{y'}m_z$	$2_z/m_z$	$2_z/m_z$	$m_{y'}*$	$m_{x'}*m_{y'}*m_z$	
$m_{xy}m_{\bar{x}y}m_z$	m_{xz} , 4_y	$m_z\bar{3}_pm_{xy}$	$m_xm_{yz}m_{\bar{y}z}$	$\bar{1}$	$m_{xz}*$	$2_{xz}*/m_{xz}*$	$[l2h\bar{l}]$
$\bar{1}$	m_{xz} , 2_{xz}	$2_{xz}/m_{xz}$	$\bar{1}$	$\bar{1}$	$m_{xz}*$	$2_{xz}*/m_{xz}*$	
4_z	2_{xz} , 4_y	$4_z3_p2_{xy}$	4_x	1	$2_{xz}*$	$2_{xz}*$	[010]
1	2_{xz}	2_{xz}	1	1	$2_{xz}*$	$2_{xz}*$	
4_z	m_{xz} , $\bar{4}_y$	$m_z\bar{3}_pm_{xy}$	4_x	1	$m_{xz}*$	$m_{xz}*$	[010]
1	m_{xz}	m_{xz}	1	1	$m_{xz}*$	$m_{xz}*$	
$\bar{4}_z$	m_{xz} , $\bar{4}_y$	$\bar{4}_z3_pm_{xy}$	$\bar{4}_x$	1	$m_{xz}*$	$m_{xz}*$	[010]
1	m_{xz}	m_{xz}	1	1	$m_{xz}*$	$m_{xz}*$	
$\bar{4}_z$	2_{xz} , 4_y	$m_z\bar{3}_pm_{xy}$	$\bar{4}_x$	1	$2_{xz}*$	$2_{xz}*$	[010]
1	2_{xz}	2_{xz}	1	1	$2_{xz}*$	$2_{xz}*$	
$4_z/m_z$	m_{xz} , 4_y	$m_z\bar{3}_pm_{xy}$	$4_x/m_x$	$\bar{1}$	$m_{xz}*$	$2_{xz}*/m_{xz}*$	[010]

DOMAIN PAIR SYMMETRY REDUCTION

$\mathbf{\bar{1}}$	$2_{xz}, m_{xz}$	$\mathbf{2_{xz}/m_{xz}}$	$\mathbf{\bar{1}}$	$\mathbf{\bar{1}}$	m_{xz}^{*}	$\mathbf{2_{xz}^{*}/m_{xz}^{*}}$	
$\mathbf{4_z2_x2_{xy}}$	$2_{xz}, 2_{\bar{x}z}$	$\mathbf{4_z3_p2_{xy}}$	$\mathbf{4_x2_y2_{yz}}$	$\mathbf{2_x2_y2_z}$	2_{xz}^{*}	$\mathbf{4_y^{*}2_x2_{xz}^{*}}$	[010]
$\mathbf{2_y}$	$2_{xz}, 2_{\bar{x}z}$	$\mathbf{2_{xz}2_y2_{\bar{x}z}}$	$\mathbf{2_y}$	$\mathbf{2_y}$	2_{xz}^{*}	$\mathbf{2_{xz}^{*}2_y2_{\bar{x}z}^{*}}$	
$\mathbf{4_z2_x2_{xy}}$	$m_{xz}, m_{\bar{x}z}$	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{4_x2_y2_{yz}}$	$\mathbf{2_x2_y2_z}$	m_{xz}^{*}	$\mathbf{\bar{4}_y^{*}2_zm_{xz}^{*}}$	[010]
$\mathbf{2_y}$	$m_{xz}, m_{\bar{x}z}$	$\mathbf{m_{xz}2_ym_{\bar{x}z}}$	$\mathbf{2_y}$	$\mathbf{2_y}$	m_{xz}^{*}	$\mathbf{m_{xz}^{*}2_ym_{\bar{x}z}^{*}}$	
$\mathbf{4_zm_xm_{xy}}$	$2_{xz}, m_{\bar{x}z}$	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{4_xm_ym_{yz}}$	$\mathbf{m_y}$	2_{xz}^{*}	$\mathbf{2_{xz}^{*}m_ym_{\bar{x}z}^{*}}$	[010]
$\mathbf{m_y}$	$2_{xz}, m_{\bar{x}z}$	$\mathbf{2_{xz}m_ym_{\bar{x}z}}$	$\mathbf{m_y}$	$\mathbf{m_y}$	2_{xz}^{*}	$\mathbf{2_{xz}^{*}m_ym_{\bar{x}z}^{*}}$	
$\mathbf{\bar{4}_z2_xm_{xy}}$	$m_{xz}, m_{\bar{x}z}$	$\mathbf{\bar{4}_z3_pm_{xy}}$	$\mathbf{\bar{4}_x2_zm_{yz}}$	$\mathbf{2_x2_y2_z}$	m_{xz}^{*}	$\mathbf{\bar{4}_y^{*}2_zm_{xz}^{*}}$	[010]
$\mathbf{2_y}$	$m_{xz}, m_{\bar{x}z}$	$\mathbf{m_{xz}2_ym_{\bar{x}z}}$	$\mathbf{2_y}$	$\mathbf{2_y}$	m_{xz}^{*}	$\mathbf{m_{xz}^{*}2_ym_{\bar{x}z}^{*}}$	
$\mathbf{\bar{4}_zm_x2_{xy}}$	$2_{\bar{x}z}, m_{xz}$	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{\bar{4}_xm_z2_{yz}}$	$\mathbf{m_y}$	m_{xz}^{*}	$\mathbf{m_{xz}^{*}m_y2_{\bar{x}z}^{*}}$	[010]
$\mathbf{m_y}$	$2_{\bar{x}z}, m_{xz}$	$\mathbf{m_{xz}m_y2_{\bar{x}z}}$	$\mathbf{m_y}$	$\mathbf{m_y}$	m_{xz}^{*}	$\mathbf{m_{xz}^{*}m_y2_{\bar{x}z}^{*}}$	
$\mathbf{\bar{4}_z2_xm_{xy}}$	$2_{xz}, 2_{\bar{x}z}$	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{\bar{4}_x2_zm_{yz}}$	$\mathbf{2_x2_y2_z}$	2_{xz}^{*}	$\mathbf{4_y^{*}2_x2_{xz}^{*}}$	[010]
$\mathbf{2_y}$	$2_{xz}, 2_{\bar{x}z}$	$\mathbf{2_{xz}2_y2_{\bar{x}z}}$	$\mathbf{2_y}$	$\mathbf{2_y}$	2_{xz}^{*}	$\mathbf{2_{xz}^{*}2_y2_{\bar{x}z}^{*}}$	
$\mathbf{4_z/m_zm_xm_{xy}}$	$m_{xz}, m_{\bar{x}z}$	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{4_x/m_xm_ym_{yz}}$	$\mathbf{m_xm_ym_z}$	m_{xz}^{*}	$\mathbf{4_y^{*}/m_ym_xm_{xz}^{*}}$	[010]
$\mathbf{2_y/m_y}$	$m_{xz}, m_{\bar{x}z}$	$\mathbf{m_{xz}m_ym_{\bar{x}z}}$	$\mathbf{2_y/m_y}$	$\mathbf{2_y/m_y}$	m_{xz}^{*}	$\mathbf{m_{xz}^{*}m_ym_{\bar{x}z}^{*}}$	
$\mathbf{3_p}$	$2_x, 3_r$	$\mathbf{2_z3_p}$	$\mathbf{3_r}$	$\mathbf{1}$	2_x^{*}	$\mathbf{2_x^{*}}$	$[01\bar{1}]$
$\mathbf{1}$	2_x	$\mathbf{2_x}$	$\mathbf{1}$	$\mathbf{1}$	2_x^{*}	$\mathbf{2_x^{*}}$	
$\mathbf{3_p}$	$m_x, \bar{3}_r$	$\mathbf{m_z\bar{3}_p}$	$\mathbf{3_r}$	$\mathbf{1}$	m_x^{*}	$\mathbf{m_x^{*}}$	$[01\bar{1}]$
$\mathbf{1}$	m_x	$\mathbf{m_x}$	$\mathbf{1}$	$\mathbf{1}$	m_x^{*}	$\mathbf{m_x^{*}}$	
$\mathbf{3_p}$	$2_{xy}, 4_y$	$\mathbf{4_z3_p2_{xy}}$	$\mathbf{3_r}$	$\mathbf{1}$	2_{xy}^{*}	$\mathbf{2_{xy}^{*}}$	$[1\bar{1}0]$
$\mathbf{1}$	2_{xy}	$\mathbf{2_{xy}}$	$\mathbf{1}$	$\mathbf{1}$	2_{xy}^{*}	$\mathbf{2_{xy}^{*}}$	
$\mathbf{3_p}$	$m_{xy}, \bar{4}_y$	$\mathbf{\bar{4}_z3_pm_{xy}}$	$\mathbf{3_r}$	$\mathbf{1}$	m_{xy}^{*}	$\mathbf{m_{xy}^{*}}$	$[1\bar{1}0]$
$\mathbf{1}$	m_{xy}	$\mathbf{m_{xy}}$	$\mathbf{1}$	$\mathbf{1}$	m_{xy}^{*}	$\mathbf{m_{xy}^{*}}$	
$\mathbf{\bar{3}_p}$	$m_x, 3_r$	$\mathbf{m_z\bar{3}_p}$	$\mathbf{\bar{3}_r}$	$\mathbf{\bar{1}}$	m_x^{*}	$\mathbf{2_x^{*}/m_x^{*}}$	$[01\bar{1}]$
$\mathbf{\bar{1}}$	$2_x, m_x$	$\mathbf{2_x/m_x}$	$\mathbf{\bar{1}}$	$\mathbf{\bar{1}}$	m_x^{*}	$\mathbf{2_x^{*}/m_x^{*}}$	
$\mathbf{\bar{3}_p}$	$m_{xy}, 4_y$	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{\bar{3}_r}$	$\mathbf{\bar{1}}$	2_{xy}^{*}	$\mathbf{2_{xy}^{*}/m_{xy}^{*}}$	$[1\bar{1}0]$
$\mathbf{\bar{1}}$	$2_{xy}, m_{xy}$	$\mathbf{2_{xy}/m_{xy}}$	$\mathbf{\bar{1}}$	$\mathbf{\bar{1}}$	2_{xy}^{*}	$\mathbf{2_{xy}^{*}/m_{xy}^{*}}$	
$\mathbf{3_p2_{\bar{x}y}}$	$2_x, 2_{yz}$	$\mathbf{4_z3_p2_{xy}}$	$\mathbf{3_r2_{xy}}$	$\mathbf{2_{\bar{y}z}}$	2_x^{*}	$\mathbf{2_x^{*}2_{yz}^{*}2_{\bar{y}z}}$	$[01\bar{1}]$
$\mathbf{2_{\bar{y}z}}$	$2_x, 2_{yz}$	$\mathbf{2_x2_{yz}2_{\bar{y}z}}$	$\mathbf{2_{\bar{y}z}}$	$\mathbf{2_{\bar{y}z}}$	2_x^{*}	$\mathbf{2_x^{*}2_{yz}^{*}2_{\bar{y}z}}$	
$\mathbf{3_p2_{\bar{x}y}}$	m_x, m_{yz}	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{3_r2_{xy}}$	$\mathbf{2_{\bar{y}z}}$	m_x^{*}	$\mathbf{m_x^{*}m_{yz}^{*}2_{\bar{y}z}}$	$[01\bar{1}]$
$\mathbf{2_{\bar{y}z}}$	m_x, m_{yz}	$\mathbf{m_xm_{yz}2_{\bar{y}z}}$	$\mathbf{2_{\bar{y}z}}$	$\mathbf{2_{\bar{y}z}}$	m_x^{*}	$\mathbf{m_x^{*}m_{yz}^{*}2_{\bar{y}z}}$	
$\mathbf{3_pm_{\bar{x}y}}$	$2_x, m_{yz}$	$\mathbf{\bar{4}_z3_pm_{xy}}$	$\mathbf{3_rm_{xy}}$	$\mathbf{m_{\bar{y}z}}$	2_x^{*}	$\mathbf{2_x^{*}m_{yz}^{*}m_{\bar{y}z}}$	$[01\bar{1}]$
$\mathbf{m_{\bar{y}z}}$	$2_x, m_{yz}$	$\mathbf{2_xm_{yz}m_{\bar{y}z}}$	$\mathbf{m_{\bar{y}z}}$	$\mathbf{m_{\bar{y}z}}$	2_x^{*}	$\mathbf{2_x^{*}m_{yz}^{*}m_{\bar{y}z}}$	
$\mathbf{3_pm_{\bar{x}y}}$	$m_x, 2_{yz}$	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{3_rm_{xy}}$	$\mathbf{m_{\bar{y}z}}$	2_{yz}^{*}	$\mathbf{m_x^{*}2_{yz}^{*}m_{\bar{y}z}}$	$[01\bar{1}]$
$\mathbf{m_{\bar{y}z}}$	$m_x, 2_{yz}$	$\mathbf{m_x2_{yz}m_{\bar{y}z}}$	$\mathbf{m_{\bar{y}z}}$	$\mathbf{m_{\bar{y}z}}$	2_{yz}^{*}	$\mathbf{m_x^{*}2_{yz}^{*}m_{\bar{y}z}}$	
$\mathbf{\bar{3}_pm_{\bar{x}y}}$	m_x, m_{yz}	$\mathbf{m_z\bar{3}_pm_{xy}}$	$\mathbf{\bar{3}_rm_{xy}}$	$\mathbf{2_{\bar{y}z}/m_{\bar{y}z}}$	m_x^{*}	$\mathbf{m_x^{*}m_{yz}^{*}m_{\bar{y}z}}$	$[01\bar{1}]$
$\mathbf{2_{\bar{y}z}/m_{\bar{y}z}}$	m_x, m_{yz}	$\mathbf{m_xm_{yz}m_{\bar{y}z}}$	$\mathbf{2_{\bar{y}z}/m_{\bar{y}z}}$	$\mathbf{2_{\bar{y}z}/m_{\bar{y}z}}$	m_x^{*}	$\mathbf{m_x^{*}m_{yz}^{*}m_{\bar{y}z}}$	